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Contractions of AdS brane algebra and superGalileon Lagrangians

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Abstract

We examine AdS Galileon Lagrangians using the method of non-linear realization. By contractions 1) flat curvature limit, 2) non-relativistic brane algebra limit and 3) (1)+(2) limits we obtain DBI, Newton-Hooke and Galilean Galileons respectively. We make clear how these Lagrangians appear as invariant 4-forms and/or pseudo-invariant Wess-Zumino terms using Maurer-Cartan equations on the coset $G/SO(3,1)$. We show the equations of motion are written in terms of the MC forms only and explain why the inverse Higgs condition is obtained as the equation of motion for all cases.

The supersymmetric extension is also examined using a supercoset $SU(2,2|1)/(SO(3,1) \times U(1))$ and five WZ forms are constructed. They are reduced to the corresponding five Galileon WZ forms in the bosonic limit and are candidates for supersymmetric Galileon action.

keywords: Galileon, non-linear realization, supersymmetry

1 Introduction

Modifications of gravity using higher dimensional non-compact extra dimensions are important approaches to solve cosmological problems [1]. The Galileons appear in such context [2] and are interesting both in theoretical as well as phenomenological applications. (See for example recent reviews [3][4]).

It has been shown the Galileon actions are obtained based on the non-relativistic 3-brane algebra in 5 dimensions. It is the 5-dimensional Poincare algebra in which non-relativistic limit is taken in the transverse fifth direction.¹ The Galileon appears as a Goldstone scalar field in the broken transverse direction of the 3-brane and is satisfying second order equation of motion [7]. It has been clarified [8] that 5 possible forms of Galileon Lagrangians are the WZ Lagrangians constructed from closed and non-trivial 5-forms on the group manifold. It was also shown there is only one WZ Lagrangian for the DBI Galileon [9][10] and conformal Galileon [11][8] theories.

The Galileon Lagrangians are constructed [8] using the method of non-linear realization [12] for space-time symmetry algebras [13]. The Maurer-Cartan(MC) one forms on the coset G/H are the building blocks of the Lagrangians. Here G is the brane algebra and $H = SO(3, 1)$ is the unbroken longitudinal Lorentz algebra of the brane. Galileons appear as the Goldstone mode with respect to the broken transverse translation. The G -invariant Lagrangians are constructed from either H -invariant 4 forms or pseudo-invariant 4-forms which are obtained from closed and Chevalley-Eilenberg(CE) non-trivial H -invariant 5 forms as the WZ Lagrangians [14, 15].

In this paper we restrict models of single Galileon and begin with the AdS algebra in 5 dimensions for AdS Galileon [16]. This algebra allows three contractions giving four Galileon models;

- ★ no-contraction gives AdS Galileon ,
- ★ non-relativistic brane algebra limit gives Newton-Hooke(NH) Galileons
- ★ flat curvature limit gives Poincaré(DBI) Galileons
- ★ non-relativistic and flat curvature limits gives Galilean Galileons.

$$\begin{array}{ccc}
 \text{AdS} & \xRightarrow{R \rightarrow \infty} & \text{Poincaré(DBI)} \\
 \downarrow_{\omega \rightarrow \infty} & & \downarrow_{\omega \rightarrow \infty} \\
 \text{Newton-Hooke} & \xRightarrow{R \rightarrow \infty} & \text{Galilei}
 \end{array}$$

Table 1: The contractions of AdS algebra

They are examined in detail in [8] and we extend the analysis to make clear systematically how five possible Lagrangians appear either invariant 4 forms or pseudo-invariant Wess-Zumino terms depending on the contractions for all cases using Maurer-Cartan equations on the coset $G/SO(3, 1)$. The equations of motion(EOM) are obtained by variations

¹The relativistic and non-relativistic brane actions was constructed using the non-linear realization of the brane algebras. See for example [5][6] and references therein.

of the MC forms and we can express the EOM in terms of MC forms only, without using explicit parametrization of the coset.

One purpose of this paper is to understand why the Galileons constructed in the non-linear realization satisfy at most second order equations using Maurer-Cartan equations for all four cases of Table 1. It comes from two facts; first is that the inverse Higgs condition, which eliminates the Goldstone boost vector variables in terms of the Galileon scalar, is derived as a EOM from the covariance. Second is the EOM for Galileon field is a (pullback of) sum of five invariant 4 forms and becomes at most second order differential equation for the Galileon scalar manifesting the Galileon property.

Other is to apply the method of non-linear realization to consider the supersymmetric extension of Galileon, which have been considered [17][18][19] within superfield theory using, for example superfields. We start from a superalgebra $su(2, 2|1)$ and taking Galilean limit we construct five closed invariant 4-forms and five 5-forms which reduce to the ones of bosonic Galilean Galileon in the bosonic limit. In order to obtain these five candidates for the superGalileon WZ terms we need to enlarge the superalgebra with a fermionic and central extensions.

In section 2 we make a brief review of the NLR approach of the Galileons [8] clarifying how the WZ Lagrangians appear using the Maurer Cartan (MC) equation for every cases in Table 1. In section 3 we derive the equations of motion using with variation formula of the MC forms and derive the inverse Higgs condition for every cases. The EOM for Galileons are sum of invariant 4-forms, which become second order for the Galileon scalar when the inverse Higgs condition is used. In section 4 the conformal Galileon is discussed in the same context. In section 5 the supersymmetrization of the Galileon is considered. Summary and discussions are in the final section. There are three appendices for some useful formulas. Explicit forms of MC forms, for bosonic and super cases, are also presented by choosing coset parametrizations.

2 Relativistic and Non-relativistic Brane Algebras

In this section we give a reformulation of Galileon Lagrangians [8] clarifying the "WZ" property in using the MC equation. We start with the AdS algebra in d dimensions and construct invariant $p+1$ -forms and closed and invariant $p+2$ -forms to obtain candidates of p -brane Lagrangians for Galileons.

The AdS algebra in d dimensions is $so(d-1,2)$ and is written as

$$\begin{aligned} [P_A, P_B] &= -i \frac{1}{R^2} M_{AB}, & [P_A, M_{BC}] &= -i \eta_{A[B} P_{C]}, \\ [M_{AB}, M_{CD}] &= -i \eta_{B[C} M_{AD]} + i \eta_{A[C} M_{BD]}, \end{aligned} \quad (2.1)$$

where $A, B = 0, \dots, d-1$, $\eta_{AB} = (-, +, \dots, +)$ and R is the radius of AdS. In $R \rightarrow \infty$ limit it is contracted to the Poincare algebra. In the presence of p -brane we split the space-time indices $A = 0, \dots, d-1$ into the longitudinal directions $a = 0, 1, \dots, p$ and transverse one $a' = p+1, \dots, d-1$. The Newton Hooke(NH) algebra [20] for non-relativistic brane [21] is that in which the light velocity goes to infinity in the transverse directions. We get it

by a contraction of the AdS algebra (2.1) using a rescaling

$$P_{a'} \rightarrow \omega P_{a'}, \quad M_{aa'} \rightarrow \omega M_{aa'} \quad (2.2)$$

and $\omega \rightarrow \infty$. In this paper we apply it to single Galileon models and we restrict one transverse direction, i.e. p -brane in $d = p + 2$ dimensions. Writing the single transverse index as $a' = \pi$ and the boost generators in the transverse direction as $M_{a\pi} = B_a$ the algebra becomes

$$\begin{aligned} [P_a, P_b] &= -i \frac{1}{R^2} M_{ab}, & [P_a, P_\pi] &= -\frac{i}{R^2} B_a, & [P_a, M_{cd}] &= -i \eta_{a[c} P_{d]}, \\ [B_a, P_b] &= i \eta_{ab} P_\pi, & [B_a, P_\pi] &= -\frac{i}{\omega^2} P_a, & [B_a, B_b] &= \frac{i}{\omega^2} M_{ab}, \\ [B_a, M_{cd}] &= -i \eta_{a[c} B_{d]}, & [M_{ab}, M_{cd}] &= -i \eta_{b[c} M_{ad]} + i \eta_{a[c} M_{bd]}. \end{aligned} \quad (2.3)$$

In the $R \rightarrow \infty$ limit it becomes the Poincaré (DBI) brane algebra and in the $\omega \rightarrow \infty$ it goes to NH brane algebra. Taking both $\omega \rightarrow \infty$ and $R \rightarrow \infty$ limits it becomes Galilean brane algebra (Galileon algebra). We consider these four cases by comparison. (Table 1)

Taking the AdS algebra G in (2.3) and the stability group H as the longitudinal Lorentz algebra $so(p, 1)$ we describe the system using a coset G/H . The Maurer-Cartan(MC) form Ω is introduced by

$$\Omega = -ig^{-1}dg = G_A L^A = P_a L_P^a + \frac{1}{2} M_{ab} L^{ab} + P_\pi L^\pi + B_a L_B^a, \quad g \in G/H. \quad (2.4)$$

Using the first expression of $\Omega = -ig^{-1}dg$ it holds identically the MC equation

$$d\Omega + i\Omega \wedge \Omega = 0. \quad (2.5)$$

Using the second expression of (2.4), $\Omega = G_A L^A$, and for algebra $[G_A, G_B] = if_{AB}^C G_C$ (2.5) gives MC equation for the component one forms L^A 's as

$$dL^A + \frac{1}{2} f_{BC}^A L^C \wedge L^B = 0. \quad (2.6)$$

For the AdS algebra (2.3) the MC equation becomes²

$$\begin{aligned} dL_P^a &+ L_P^c L_c^a + \frac{1}{\omega^2} L_B^a L^\pi = 0, \\ dL^\pi &+ L_P^c L_{Bc} = 0, \\ dL^{ab} &+ L^{ac} L_c^b - \frac{1}{\omega^2} L_B^a L_B^b + \frac{1}{R^2} L_P^a L_P^b = 0, \\ dL_B^a &+ L^{ac} L_{Bc} + \frac{1}{R^2} L_P^a L^\pi = 0. \end{aligned} \quad (2.7)$$

The consistency(integrability) of the set of MC equations (2.7) is equivalent to holds the Jacobi identities of the algebra (2.3). (See Appendix A where some useful formulas are summarized.)

² We often abbreviate " \wedge " symbol for wedge products.

In the non-linear realization on the coset G/H the coset elements g 's are parametrized by coset coordinates Z^M . Under infinitesimal global G transformations the coset element g transforms as

$$g(Z) \rightarrow g' = g_\epsilon g(Z) h^{-1}(\epsilon, Z) = g(Z'), \quad g_\epsilon \in G, \quad h \in H, \quad (2.8)$$

where $h(\epsilon, Z)$ is the compensating local H transformation so that g' becomes a coset element. The MC form Ω transforms under G as

$$\Omega \rightarrow \Omega' = h \Omega h^{-1} - i h d h^{-1}. \quad (2.9)$$

Since the last term belongs to the subalgebra H one forms L 's associated with G/H transform as H covariants and L 's associated with H transform as H gauge connections. Now L^π is a scalar, L_P^a and L_B^a are $so(p, 1)$ vectors and L^{ab} is a gauge connection of the $so(p, 1)$ under G transformations. The G -invariant p -brane action can be constructed from local H (thus $so(p, 1)$ Lorentz) invariant $p+1$ forms. In addition when closed and local H -invariant $p+2$ forms exist and are CE non-trivial, that is they are not written as "d" of some $p+1$ forms from L 's, we can construct WZ Lagrangians that is pseudo invariant under G transformations from them [14][15].

We can construct H -invariant $p+1$ forms K_q , ($q = 0, 1, \dots, p+1$) from wedge products of q vectors L_P^a and $(p+1-q)$ vectors L_B^a ,

$$K_q \equiv \epsilon_{a_0 \dots a_{q-1} a_q \dots a_p} L_P^{a_0} \dots L_P^{a_{q-1}} L_B^{a_q} \dots L_B^{a_p}, \quad (q = 0, \dots, p+1), \quad (2.10)$$

where $\epsilon_{a_0 \dots a_p}$ is the Levi-Civita tensor with $\epsilon_{0 \dots p} = +1$. K_q 's are only possible non-trivial invariant $p+1$ forms constructed from wedge product of the MC one forms.³ Taking wedge product with H -scalar L^π we define H -invariant $p+2$ forms Ω_q as

$$\Omega_q = K_q L^\pi, \quad (q = 0, \dots, p+1). \quad (2.11)$$

They are only possible H -invariant $p+2$ forms using wedge products of L 's.

Using the MC equations (2.7) they are related by

$$dK_q = -\frac{(-)^p}{\omega^2} q \Omega_{q-1} - \frac{(-)^p}{R^2} (p+1-q) \Omega_{q+1}, \quad (2.12)$$

and Ω_q 's are closed,

$$d\Omega_q = dK_q L^\pi + (-)^{p+1} K_q (-L_{Pa} L_B^a) = 0. \quad (2.13)$$

The first term of (2.13) vanishes since dK_q includes L^π as in (2.12). In the second term either L_P^a or L_B^a exists in K_q for every a then it vanishes. Thus Ω_q 's are H -invariant and closed $p+2$ forms.

³ There is a H -invariant 2-form $(\eta_{ab} L_P^a \wedge L_B^b = -dL^\pi)$ contracted using η_{ab} . However a possible 4-form is exact $(\eta_{ab} L_P^a \wedge L_B^b) \wedge (\eta_{cd} L_P^c \wedge L_B^d) = dL^\pi \wedge dL^\pi$ and is not used for three brane Lagrangian in 4 dimensions.

These K_q and Ω_q are used for constructing the Galileon Lagrangians by taking pull back to the p -brane world volume [8]. K_q is a possible candidate of the G -invariant Lagrangian when it is not closed. Ω_q is used to construct G -pseudo-invariant WZ Lagrangian as $\Omega_q = d\mathcal{L}_q^{WZ}$ when Ω_q is CE non-trivial, i.e. it is not expressed in a form $d\Lambda^{p+1}$ with some H -invariant $p+1$ form Λ^{p+1} constructed from L 's. Since only possible non-trivial invariant $p+1$ forms are K_q 's Λ^{p+1} is a linear combination of K_q 's for CE-trivial Ω_q . Eq.(2.12) tells the CE cohomological structure and numbers of non-closed K_q 's and non-trivial Ω_q 's, i.e. numbers of invariant and WZ Lagrangians. (2.12) is written in a matrix form as

$$\begin{pmatrix} dK_0 \\ dK_1 \\ \vdots \\ dK_p \\ dK_{p+1} \end{pmatrix} = -(-)^p \begin{pmatrix} 0 & \frac{p+1}{R^2} & & & \\ \frac{1}{\omega^2} & 0 & \frac{p}{R^2} & & \\ & \frac{2}{\omega^2} & 0 & \frac{p-1}{R^2} & \\ & & \ddots & \ddots & \ddots \\ & & & \frac{p}{\omega^2} & 0 & \frac{1}{R^2} \\ & & & & \frac{p+1}{\omega^2} & 0 \end{pmatrix} \begin{pmatrix} \Omega_0 \\ \Omega_1 \\ \vdots \\ \Omega_p \\ \Omega_{p+1} \end{pmatrix} \quad (2.14)$$

Let r is the *rank* of the $(p+2) \times (p+2)$ matrix M appearing in (2.14), r of Ω_q are expressed in terms of dK 's then are CE trivial. The remaining $(p+2-r)$ of Ω_q are CE non-trivial and the number of WZ Lagrangians is $(p+2-r)$. It also tells there are $(p+2-r)$ independent linear combinations of dK 's that vanish identically. There remain r linear combinations of K 's that are not closed which gives r G -invariant Lagrangians. In total there are $p+2$ possible Lagrangians, r G -invariant Lagrangians and $(p+2-r)$ WZ Lagrangians.

The number of WZ Lagrangians $(p+2-r)$ is determined by the rank r of the matrix M in (2.14) and it depends on how the algebra is contracted. For the four contractions in the Table 1 ranks r of the matrix M are

$$\begin{aligned} \text{Galilean Galileon } (\omega \rightarrow \infty, R \rightarrow \infty) & : r = 0, \\ \text{DBI Galileon } (\omega = 1, R \rightarrow \infty) & : r = p+1, \\ \text{NH Galileon } (\omega \rightarrow \infty, R = 1) & : r = p+1, \\ \text{AdS Galileon (for odd } p) \text{ } (\omega = 1) & : r = p+1, \\ & \text{(for even } p) \text{ } (\omega = 1) : r = p+2. \end{aligned}$$

We will examine the possible Lagrangians for these cases in some detail.

1) Galilean brane (Galileon) $[\omega \rightarrow \infty, R \rightarrow \infty]$

(2.14) tells $M = 0$ and $r = 0$, all of the invariant $p+1$ forms K_q 's are closed in this limit $\omega \rightarrow \infty, R \rightarrow \infty$,

$$dK_q = 0, \quad (q = 0, 1, \dots, p+1). \quad (2.15)$$

If K_q 's are used in the Lagrangian they are surface terms. On the other hand closed $p+2$ forms Ω_q , $(q = 0, 1, \dots, p+1)$ are CE non-trivial and are used as WZ $p+2$ forms. There are $p+2$ WZ Lagrangians \mathcal{L}_q^{WZ} , $(q = 0, 1, \dots, p+1)$ satisfying $\Omega_q = d\mathcal{L}_q^{WZ}$ and \mathcal{L}_q^{WZ} 's are pseudo-invariant $p+2$ Galileon Lagrangians [8]. Due to CE non-triviality of Ω_q the WZ

Lagrangians \mathcal{L}_q^{WZ} are not expressed using MC forms. It requires coset coordinates to write down the WZ Lagrangians \mathcal{L}_q^{WZ} explicitly.

2) Poincaré brane (DBI Galileon) $[\omega = 1, R \rightarrow \infty]$,

In this case (2.14) shows

$$dK_0 = 0, \quad \Omega_q = -\frac{(-)^p}{(q+1)} dK_{q+1}, \quad (q = 0, 1, \dots, p). \quad (2.16)$$

K_0 is closed and is a surface term. $K_q, (q = 1, \dots, p+1)$'s are H -invariant $p+1$ forms and are used as the Lagrangians (Lovelock invariants) [22]. $\Omega_q, (q = 0, 1, \dots, p)$'s are CE trivial since they are given as "d" of MC forms K_{q+1} as in (2.16). The closed $p+2$ form $\Omega_{p+1} = K_{p+1}L^\pi$ is however non-trivial and is used as the WZ Lagrangian \mathcal{L}_{p+1}^{WZ} satisfying $\Omega_{p+1} = K_{p+1}L^\pi = d\mathcal{L}_{p+1}^{WZ}$. The only one WZ Lagrangian \mathcal{L}_{p+1}^{WZ} is the tadpole term [8] .

3) NH Galileon $[\omega \rightarrow \infty, R \text{ finite}]$.

In this case (2.14) gives

$$\Omega_{q+1} = -\frac{R^2(-)^p}{(p-q+1)} dK_q, \quad (q = 0, \dots, p), \quad dK_{p+1} = 0. \quad (2.17)$$

K_{p+1} is closed and gives a surface term (cosmological constant). $\Omega_q, (q = 1, \dots, p+1)$ are written in terms of dK_{q-1} and are CE trivial as in (2.17). There is only one CE non-trivial closed $p+2$ form Ω_0 that is used as the WZ $p+2$ form to construct WZ Lagrangian $\Omega_0 = d\mathcal{L}_0^{WZ}$.

4) AdS Galileon $[\omega = 1, R \text{ finite}]$,

In this case the rank of the matrix M in (2.14) is $p+1$ for odd p and $p+2$ for even p . Eq.(2.14) is more explicitly,

$$\begin{aligned} dK_0 &= -(-)^p \left(\frac{p+1}{R^2} \Omega_1 \right), & dK_1 &= -(-)^p \left(\Omega_0 + \frac{p}{R^2} \Omega_2 \right), \\ dK_2 &= -(-)^p \left(2\Omega_1 + \frac{p-1}{R^2} \Omega_3 \right), & \dots & \\ &\dots & dK_{p-1} &= -(-)^p \left((p-1)\Omega_{p-2} + \frac{2}{R^2} \Omega_p \right), \\ dK_p &= -(-)^p \left(p\Omega_{p-1} + \frac{1}{R^2} \Omega_{p+1} \right), & dK_{p+1} &= -(-)^p (p+1)\Omega_p. \end{aligned} \quad (2.18)$$

The first equation means Ω_1 is proportional to dK_0 and is CE trivial, the third one tells Ω_3 is also CE trivial. Then all Ω_{odd} are CE trivial. Similarly starting from the last equation $\Omega_p, \Omega_{p-2}, \dots$ are CE trivial. When p is even all $\Omega_q, (q = 0, \dots, p+1)$ can be expressed as a linear combination of dK_j then are CE trivial. On the other hand for odd p only $\Omega_q, (q = 1, 3, \dots, p)$ can be expressed as linear combinations of dK_{even} with a closure relation

$$d \left[\sum_{i=0}^{\frac{p+1}{2}} \frac{(-)^i}{R^{2i}} \frac{(p+1)!!}{(2i)!!(p+1-2i)!!} K_{2i} \right] = 0. \quad (2.19)$$

Ω_q , ($q = 0, 2, \dots, p+1$) are expressed using dK_{odd} 's and one of Ω_{even} . For example K_q , ($q = 0, \dots, p$) can be taken as independent non-trivial invariant $p+1$ forms and Ω_{p+1} is used to construct the WZ Lagrangian \mathcal{L}_{p+1}^{WZ} . K_{p+1} is a linear combination of other K_{even} up to closed form due to (2.19) and Ω_q , ($q = 0, \dots, p$) are expressed in terms of dK_q 's and Ω_{p+1} .

In summary Galileon Lagrangians are given by taking pullback of these forms,⁴

1) Galilean brane, $[\omega \rightarrow \infty, R \rightarrow \infty]$,

$$\mathcal{L}^{Gal} = \sum_{q=0}^{p+1} b^q \mathcal{L}_q^{WZ}. \quad (2.20)$$

2) Poincare brane (DBI), $[\omega = 1, R \rightarrow \infty]$,

$$\mathcal{L}^{Poincare} = \sum_{q=1}^{p+1} c^q K_q + b^{p+1} \mathcal{L}_{p+1}^{WZ}. \quad (2.21)$$

3) NH Galileon, $[\omega \rightarrow \infty, R \text{ finite}]$,

$$\mathcal{L}^{NH} = \sum_{q=0}^p c^q K_q + b^0 \mathcal{L}_0^{WZ}. \quad (2.22)$$

4) AdS Galileon, $[\omega = 1, R \text{ finite}]$,

$$\mathcal{L}^{AdS} = \sum_{q=0}^{p+1} c^q K_q, \quad (\text{for even } p), \quad (2.23)$$

$$\mathcal{L}^{AdS} = \sum_{q=0}^p c^q K_q + b^{p+1} \mathcal{L}_{p+1}^{WZ}, \quad (\text{for odd } p). \quad (2.24)$$

Each Lagrangian has $p+2$ independent terms, apart from surface terms. They are formally written as

$$\mathcal{L}^{tot} = \sum_{q=0}^{p+1} [c^q K_q + b^q \mathcal{L}_q^{WZ}], \quad (2.25)$$

where only $p+2$ of coefficients c^q and b^q are non-vanishing depending on the cases as above.

Gal	DBI	NH	AdS even p	AdS odd p
\mathcal{L}_0^{WZ}		$K_0 \mathcal{L}_0^{WZ}$	K_0	$K_0 \circ$
\mathcal{L}_1^{WZ}	K_1	K_1	K_1	K_1
\mathcal{L}_2^{WZ}	K_2	K_2	K_2	$K_2 \circ$
\vdots	\vdots	\vdots	\vdots	\vdots
\mathcal{L}_{p-1}^{WZ}	K_{p-1}	K_{p-1}	K_{p-1}	$K_{p-1} \circ$
\mathcal{L}_p^{WZ}	K_p	K_p	K_p	K_p
\mathcal{L}_{p+1}^{WZ}	$K_{p+1} \mathcal{L}_{p+1}^{WZ}$		K_{p+1}	$\circ \mathcal{L}_{p+1}^{WZ}$
0 $p+2$	$p+1$ 1	$p+1$ 1	$p+2$ 0	$p+1$ 1

⁴ Lagrangians are pullbacks of forms to the world-volume. The pullback notation, $L^a \rightarrow L^{a*}$, etc., is omitted for simplicity.

Table 2: Possible invariant and WZ Lagrangians. In the last line numbers of invariant Lagrangian (left) and number of WZ Lagrangian (right) are tabulated. There are ambiguity in AdS(odd p) case. Terms indicated by \circ could appear but are dependent.

3 Equations of motion

In this section we derive the equations of motion (EOM) by taking the variation of the Lagrangians with respect to the coordinates of the coset. Since the Lagrangians are constructed from MC forms not only variations of the invariant Lagrangians but also those of the WZ Lagrangians are expressed in terms of MC forms. Especially we get the inverse Higgs condition $L^\pi = 0$ as a result of the EOM for all cases.

Variations of MC forms, under any variation of the coset coordinates Z^M , is given by (A.8),

$$\delta L^A = d[\delta Z]^A + f_{BC}^A L^C [\delta Z]^B \quad (3.1)$$

where f_{BC}^A is the structure constants of the algebra and $[\delta Z]^A$ is defined by replacing dZ^M with δZ^M in the MC form L^A ,

$$[\delta Z]^A \equiv \delta Z^M L_M^A \quad \text{for} \quad L^A = dZ^M L_M^A. \quad (3.2)$$

An advantage of using (3.1) is that it does not depend on how the coset is parametrized. In the present case the coset coordinates Z^M are x^a, π, v^a associating to the G/H generators, P_a, P_π, B_a respectively. $[\delta Z]_P^a, [\delta Z]^\pi, [\delta Z]_B^a$ are L_P^a, L^π, L_B^a in which $dx^a, d\pi, dv^a$ are replaced by $\delta x^a, \delta\pi, \delta v^a$ respectively. (3.1) becomes

$$\begin{aligned} \delta L_P^a &= d[\delta Z]_P^a + L_P^c [\delta Z]_c^a - [\delta Z]_P^c L_c^a + \frac{1}{\omega^2} L_B^a [\delta Z]^\pi - \frac{1}{\omega^2} [\delta Z]_B^a L^\pi, \\ \delta L^\pi &= d[\delta Z]^\pi + L_P^c [\delta Z]_{Bc} - [\delta Z]_P^c L_{Bc}, \\ \delta L^{ab} &= d[\delta Z]^{ab} + L^{[ac} [\delta Z]_c^{b]} - \frac{1}{\omega^2} L_B^{[a} [\delta Z]_B^{b]} + \frac{1}{R^2} L_P^{[a} [\delta Z]_P^{b]}, \\ \delta L_B^a &= d[\delta Z]_B^a + L^{ac} [\delta Z]_{Bc} - [\delta Z]^{ac} L_{Bc} + \frac{1}{R^2} L_P^a [\delta Z]^\pi - \frac{1}{R^2} [\delta Z]_P^a L^\pi. \end{aligned} \quad (3.3)$$

Using it we compute variations of K_q and \mathcal{L}_q^{WZ} under general variations (3.3). For K_q , apart from exact forms,

$$\begin{aligned} \delta K_q &= - \left\{ \frac{1}{\omega^2} q(p-q+2) \epsilon_{a_0 \dots a_{q-2} a_{q-1} \dots a_p} L_P^{a_0} \dots L_P^{a_{q-2}} L_B^{a_{q-1}} \dots L_B^{a_{p-1}} \right. \\ &\quad \left. + \frac{1}{R^2} (p-q)(p-q+1) \epsilon_{a_0 \dots a_q a_{q+1} \dots a_p} L_P^{a_0} \dots L_P^{a_q} L_B^{a_{q+1}} \dots L_B^{a_{p-1}} \right\} L^\pi [\delta Z]_B^{a_p} \\ &\quad - \left\{ \frac{1}{\omega^2} q(q-1) \epsilon_{a_0 \dots a_{q-3} a_{q-2} \dots a_p} L_P^{a_0} \dots L_P^{a_{q-3}} L_B^{a_{q-2}} \dots L_B^{a_{p-1}} \right. \\ &\quad \left. + \frac{1}{R^2} (q+1)(p-q+1) \epsilon_{a_0 \dots a_{q-1} a_q \dots a_p} L_P^{a_0} \dots L_P^{a_{q-1}} L_B^{a_q} \dots L_B^{a_{p-1}} \right\} L^\pi [\delta Z]_P^{a_p}, \\ &\quad + \left\{ \frac{q}{\omega^2} K_{q-1} + \frac{(p-q+1)}{R^2} K_{q+1} \right\} [\delta Z]^\pi. \end{aligned} \quad (3.4)$$

Since the WZ Lagrangian \mathcal{L}_q^{WZ} is defined from the closed form as $\Omega_q = d\mathcal{L}_q^{WZ}$ the variation of \mathcal{L}_q^{WZ} is determined from that of Ω_q . Actually $\delta\Omega_q$ is written in an exact form and $\delta\mathcal{L}_q^{WZ}$ is read from $\delta\Omega_q = d[\delta\mathcal{L}_q^{WZ}]$, up to closed form, as

$$\begin{aligned}\delta\mathcal{L}_q^{WZ} &= (-)^p(p-q+1) \epsilon_{a_0\dots a_{q-1}a_q\dots a_p} L_P^{a_0}\dots L_P^{a_{q-1}} L_B^{a_q}\dots L_B^{a_{p-1}} L^\pi [\delta Z]_B^{a_p} \\ &+ (-)^p q \epsilon_{a_0\dots a_{q-2}a_{q-1}\dots a_p} L_P^{a_0}\dots L_P^{a_{q-2}} L_B^{a_{q-1}}\dots L_B^{a_{p-1}} L^\pi [\delta Z]_P^{a_p} \\ &- (-)^p K_q [\delta Z]^\pi.\end{aligned}\tag{3.5}$$

Note there appears no $[\delta Z]^{ab}$ term in δK_q and $\delta\mathcal{L}_q^{WZ}$ due to local $SO(p,1)$ invariance.

In deriving the Euler-Lagrange(EL) equations we take variation of the Lagrangians with respect to the coset coordinates $\{x^a, \pi, v^a\}$ associating to the G/H generators, $\{P_a, P_\pi, B_a\}$. In above $\{[\delta Z]_P^a, [\delta Z]^\pi, [\delta Z]_B^a\}$ are $\{L^a, L^\pi, L_B^a\}$ in which $\{dx^a, d\pi, dv^a\}$ are replaced by $\{\delta x^a, \delta\pi, \delta v^a\}$ respectively. Since $\{x^a, \pi, v^a\}$ are a parametrization of the coset $\{[\delta Z]_P^a, [\delta Z]^\pi, [\delta Z]_B^a\}$ and $\{\delta x^a, \delta\pi, \delta v^a\}$ are linearly related by a non-singular matrix. The Euler-Lagrange equations are thus coefficients of $\{[\delta Z]_P^a, [\delta Z]^\pi, [\delta Z]_B^a\}$ in the variations of the Lagrangians (3.4) and (3.5). It is important to notice the EL equations are written in terms of (pullback of) exterior products of $p+1$ MC 1-forms. It is contrasted with the fact that explicit forms of the WZ Lagrangians can not be written in terms of MC forms but require coset coordinates due to CE non-triviality of WZ $p+2$ -forms Ω_q .

Now the Lagrangians (2.21)-(2.24) are linear combinations of K_q and \mathcal{L}_q^{WZ} as (2.25) coefficients of $[\delta Z]_B^a$ in δK_q and $\delta\mathcal{L}_q^{WZ}$ have common factor L^π and

$$\delta\mathcal{L} = (p\text{-form})_a \wedge L^\pi [\delta Z]_B^a = 0.\tag{3.6}$$

It gives $p+1$ ($a = 0, \dots, p$) components independent EL equations and is solved by

$$L^\pi = 0,\tag{3.7}$$

which is a one-form equation in $p+1$ dimensions thus including $p+1$ independent components. The equation (3.7) is known as the inverse Higgs condition[23] either obtained as EOM or imposed in models using non-linear realization. In the present case it is derived as the EL equations from the variation of the boost coordinates v^a . We can understand the reason why L^π appears in the variation of $p+1$ forms K_q in (3.4) and \mathcal{L}_q^{WZ} in (3.5) as the common factor. Each coefficients of $[\delta Z]_B^a$ in δK_q and $\delta\mathcal{L}_q^{WZ}$ is $p+1$ form and H -vector with the index a . To construct such term from the MC forms, $L^a, L^{a\pi}$ and L^π , it is required to use one scalar one form L^π in addition to p of L_P^a, L_B^a from the H -covariance.

The same argument is applied in the variation of x^a , the coefficients of $[\delta Z]_P^a$ in the variations (3.4) and (3.5) have the common factor L^π . Then it does not give independent EOM. The fact that they are dependent is a reflection of the diffeomorphism invariance which is manifestly assured in the differential form description. We could take the coset parameters x^a associating to P_a non dynamically.

The $[\delta Z]^\pi$ term in the variation of the total Lagrangian \mathcal{L}^{tot} in (2.25) gives the EL equation

$$\sum_{q=0}^{p+1} \left[c^q \left(\frac{q}{\omega^2} K_{q-1} + \frac{(p-q+1)}{R^2} K_{q+1} \right) + b^q (-)^p K_q \right] = 0,\tag{3.8}$$

where $p+2$ of coefficients c^q and b^q vanish identically depending on the cases in (2.21)-(2.24). It is noted that the EOM (3.8) is a linear combination of all K_q , ($q = 0, \dots, p+1$) for every cases. For example for DBI case ($R \rightarrow \infty$) (3.8) is

$$c^1 K_0 + 2 c^2 K_1 + \dots + (p+1) c^{p+1} K_p + b^{p+1} (-)^p K_{q+1} = 0. \quad (3.9)$$

The inverse Higgs condition (3.7), by taking pullback to the p -brane world volume, is a set of algebraic equations determining the coset coordinates v^a associating to B_a in terms of π , the coset coordinate associating to P_π , and its first order derivatives, (see Appendix B for explicit forms),

$$L^\pi = 0, \quad \rightarrow \quad v^a = v^a(\pi, \partial\pi). \quad (3.10)$$

Using it in (3.8) it becomes EOM for π . Since L 's includes v^a at most first order derivatives, the EOM (3.8), after eliminating v^a in terms of π is at most second order differential equation of π . Then π is the Galileon field verifying second order EOM. Note we have not used any particular choice of the coset parametrization and above results are general ones.

One could use (3.10) in the Lagrangian $\mathcal{L}^{tot}(x^a, v^a, \pi)$ in (2.25) to define effective Lagrangian $\mathcal{L}^\pi(x^a, \pi)$. Although it depends on the second order derivatives of π the EL equation for π is not higher order but remains to be second order one. It is equivalent to one obtained from (3.8) using (3.10) since v^a is solved algebraically as in (3.10).

In Appendix B we solve the IH equation (3.10) in a parametrization of the coset G/H and find expressions of MC forms in each cases.

4 Conformal Galileon

In above we have considered AdS and its contracted Galileons. Here we consider briefly the conformal Galileons [11] in a similar manner as the previous sections. The conformal algebra in 4-dimensions is $so(4,2)$ and is

$$\begin{aligned} [P_a, K_b] &= +2i M_{ab} - 2i \eta_{ab} D, & [P_a, D] &= -i P_a, & [K_a, D] &= i K_a, \\ [M_{ab}, D] &= 0, & [M_{ab}, M_{cd}] &= -i \eta_{b[c} M_{ad]} + i \eta_{a[\rho} M_{b\sigma]}, \\ [P_a, M_{cd}] &= -i \eta_{a[c} P_{d]}, & [K_a, M_{cd}] &= -i \eta_{a[c} K_{d]}. \end{aligned} \quad (4.1)$$

The MC form is

$$\Omega = P_a L_P^a + K_a L_K^a + D L_D + \frac{1}{2} M_{ab} L^{ab}, \quad (4.2)$$

and the MC equation is

$$\begin{aligned} dL^{ab} + L^{ac} L_c^b - 2 L_P^{[a} L_K^{b]} &= 0, & dL_D + 2 L_P^c L_{Kc} &= 0, \\ dL_P^a + L^{ac} L_{Pc} + L_P^a L_D &= 0, & dL_K^a + L^{ac} L_{Kc} - L_K^a L_D &= 0. \end{aligned} \quad (4.3)$$

When we consider a coset $G/H = SO(4,2)/SO(3,1) = \text{Conf}/\text{Lorentz}$ the 3-brane actions are $SO(3,1)$ invariant 4-forms or WZ action obtained from $SO(3,1)$ invariant and closed

5-forms. H -invariant 4-forms constructed from the MC one forms are

$$\begin{aligned} K_0 &= \epsilon_{abcd} L_K^a L_K^b L_K^c L_K^d, & K_1 &= \epsilon_{abcd} L_P^a L_K^b L_K^c L_K^d, & K_2 &= \epsilon_{abcd} L_P^a L_P^b L_K^c L_K^d, \\ K_3 &= \epsilon_{abcd} L_P^a L_P^b L_P^c L_K^d, & K_4 &= \epsilon_{abcd} L_P^a L_P^b L_P^c L_P^d. \end{aligned} \quad (4.4)$$

They satisfy

$$dK_q = (4 - 2q) K_q L_D \equiv (4 - 2q) \Omega_q, \quad (q = 0, 1, 2, 3, 4), \quad (4.5)$$

which counts the dilatation weight. Only K_2 is closed and a WZ Lagrangian is constructed from

$$\Omega_2 = K_2 L_D = d\mathcal{L}_2^{WZ}. \quad (4.6)$$

Other 4 invariant 5-forms $\Omega_q = K_q L_D$, ($q \neq 2$) are closed but are CE trivial. There is no other possible invariant 4-form from MC forms⁵. Then the general 3-brane Lagrangian constructed from the MC forms is

$$\mathcal{L} = \sum_{q \neq 2} c^q K_q + b^2 \mathcal{L}_2^{WZ}. \quad (4.7)$$

It gives the well known conformal Galileon Lagrangian in which the coordinate associated with D is the Galileon field π .

For general odd p -brane there appears only one WZ action $\mathcal{L}_{\frac{p+1}{2}}^{WZ}$ while no WZ action exists for even p .

Conf	$p = 3$	Conf	even p
	K_0		K_0
	K_1		K_1
	\mathcal{L}_2^{WZ}		\vdots
	K_3		K_p
	K_4		K_{p+1}

Table 3: Possible invariant and WZ Lagrangians for conformal Galileons.

The variations of K_q , ($q \neq 2$) and \mathcal{L}_2^{WZ} with respect to $[\delta Z]_K^a$ give the common factor L^D and inverse Higgs equation

$$L_D = 0 \quad (4.8)$$

follows by the same reason as discussed below equation (3.7). Those with respect to $[\delta Z]_P^a$ give the same common factor L^D and the EOM is satisfied identically due to the diffeomorphism invariance. Finally the variation of the action with respect to $[\delta Z]_D$ gives

$$\sum_{q \neq 2} c^q (2q - 4) K_q + b^2 K_2 = 0. \quad (4.9)$$

When we solve the IH equation (4.8) it gives second order differential equation for the Galileon field π , see for example in Appendix B.

⁵We may consider an invariant 4-form $(L_P^a L_{Ka})^2$ but it is exact.

5 Supersymmetrization

Galileons in the supersymmetric theories are interesting and have been examined using four dimensional chiral superfield [17] and D-brane in supergravity background [18]. Here we apply present algebraic method of the bosonic Galileons to supersymmetric case. We propose a supersymmetric Galileon algebra in 5-dimensions and find five closed and invariant 5-forms for the WZ actions. They go back to the bosonic Galileon in absence of fermionic fields.

We start from the superalgebra $su(2, 2|1)$ whose bosonic subalgebra is $so(4, 2) \times U(1)$ thus $AdS_5 \times U(1)$. In 5D minimal spinors are symplectic Majorana $U(1)$ doublet. (We basically follow the spinor notations of [24][25].) The superalgebra is, using 5D Dirac matrices γ_A , ($A = 0, 1, 2, 3, 4$), the charge conjugation matrix C and Pauli matrix σ_2 , as

$$\begin{aligned} [M_{AB}, M_{CD}] &= -i (\eta_{B[C} M_{AD]} - \eta_{A[C} M_{BD]}), \\ [M_{AB}, M_{C5}] &= -i \eta_{[BC} M_{A]5}, \quad [M_{A5}, M_{C5}] = -i M_{AC} = i \eta_{55} M_{AC}, \\ [M_{AB}, Q_\beta^k] &= -\frac{i}{2} (Q^k \gamma_{AB})_\beta, \quad [M_{A5}, Q_\beta^k] = -\frac{i}{2} (Q \gamma_A \sigma_2)_\beta^k, \quad [U, Q_\beta^k] = (Q \sigma_2)_\beta^k, \\ \{Q_\alpha^i, Q_\beta^k\} &= M_{AB} (C \gamma^{AB})_{\alpha\beta} \delta^{ik} - 2 M_{A5} (C \gamma^A \sigma_2)_{\alpha\beta}^{ik} - 3 i U (C \sigma_2)_{\alpha\beta}^{ik}. \end{aligned} \quad (5.1)$$

The bosonic generators are M_{AB}, M_{A5} , ($A, B = 0, 1, 2, 3, 4$) for $so(4, 2)$ and U for $U(1)$ and $\eta_{AB} = (-; + + + +)$, $\eta_{55} = -1$. The $N=2$ supercharges Q^i , ($i = 1, 2$) are symplectic Majorana spinors satisfying reality condition $Q^\dagger = Q B^{-1} \sigma_2$, ($B = C \gamma_0$).

The bosonic generators are rescaled for the Galilean contraction as in (2.2)

$$M_{a5} = R P_a, \quad M_{45} = R P_\pi \rightarrow \omega R P_\pi, \quad M_{a4} = B_a \rightarrow \omega B_a. \quad (5.2)$$

The supercharges Q^i are rescaled to have well defined contraction limit. They are divided using projection operators $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \gamma_4 \sigma_2)$, as $Q^\pm = Q \mathcal{P}_\pm$ ⁶. Since each Q^+ and Q^- satisfies symplectic Majorana condition they can be rescaled respectively as

$$Q^+ \rightarrow \sqrt{R} Q^+, \quad Q^- \rightarrow \omega \sqrt{R} Q^-. \quad (5.3)$$

In contrast to the bosonic case we cannot take independent limits of $R \rightarrow \infty$ and $\omega \rightarrow \infty$ but the algebra (5.1) is contracted by keeping $\omega/R = c$ a constant. The contracted algebra is, rewriting $P_a - c M_{a\pi} \rightarrow P_a$ or simply choosing $c = 0$,

$$\begin{aligned} [B_a, P_b] &= i \eta_{ab} P_\pi, \quad [P_a, M_{cd}] = -i \eta_{a[c} P_{d]}, \\ [B_a, M_{cd}] &= -i \eta_{a[c} B_{d]}, \quad [M_{ab}, M_{cd}] = -i \eta_{b[c} M_{ad]} + i \eta_{a[c} M_{bd]}, \end{aligned} \quad (5.4)$$

$$[B_a, Q_\beta^{\pm k}] = -\frac{i}{2} (Q^{\mp k} \gamma_{a4})_\beta, \quad [M_{ab}, Q_\beta^{\pm k}] = -\frac{i}{2} (Q^{\pm k} \gamma_{ab})_\beta, \quad [U, Q_\beta^{\pm k}] = \pm (Q^\pm \gamma^4)_\beta^k, \quad (5.5)$$

$$\{Q_\alpha^{+i}, Q_\beta^{+k}\} = -2 P_a (C \gamma^{a4} \mathcal{P}^+)_{\alpha\beta}^{ik}, \quad \{Q_\alpha^{+i}, Q_\beta^{-k}\} = 2 P_\pi (C \mathcal{P}^-)_{\alpha\beta}^{ik} \quad (5.6)$$

and other (anti-)commutators vanish. The bosonic subalgebra (5.4) is that of the Galilean Galileon (case 1). It includes supersubalgebra whose generators are $(P_a, M_{ab}, Q_\alpha^{+i})$ forming a $\mathcal{N} = 1$ superPoincaré algebra in four dimensions. Note the projected symplectic

⁶Here $\gamma_4 = i \gamma_{0123}$ is usual γ_5 and the projections manifest $so(3, 1)$ Lorentz invariance.

Majorana supercharge Q_α^{+i} , ($i = 1, 2$, $\alpha = 1, \dots, 4$) has 4 real degrees of freedom. Although the superalgebra (5.4)-(5.6) is a supersymmetric extension of the bosonic Galilean algebra (5.4) it is not sufficient to obtain invariant closed forms using the non-linear realization of supercoset G/H shown as below. It further requires two extensions, one is to add fermionic charge Σ^{-k} in the commutator of $[P, Q]$ as was done in case of superstring [26],

$$[P_a, Q_\beta^{+k}] = -\frac{i}{2}(\Sigma^{-k}\gamma_{a4})_\beta, \quad [M_{ab}, \Sigma_\beta^{-k}] = -\frac{i}{2}(\Sigma^{-k}\gamma_{ab})_\beta, \quad [U, \Sigma_\beta^{-k}] = -(\Sigma^{-k}\gamma^4)_\beta. \quad (5.7)$$

The other is two central charges Z, \tilde{Z} added in the anti-commutators. The second of (5.6) is replaced by

$$\begin{aligned} \{Q_\alpha^{+i}, Q_\beta^{-k}\} &= 2P_\pi(C\mathcal{P}^-)_{\alpha\beta}^{ik} + Z(C\gamma^4\mathcal{P}^-)_{\alpha\beta}^{ik}, \\ \{Q_\alpha^{+i}, \Sigma_\beta^{-k}\} &= \tilde{Z}(C\gamma^4\mathcal{P}^-)_{\alpha\beta}^{ik}. \end{aligned} \quad (5.8)$$

The left invariant MC form

$$\Omega = P_a L_P^a + \frac{1}{2} M_{ab} L^{ab} + P_\pi L^\pi + B_a L_B^a + Q_\alpha^+ L_+^\alpha + Q_\alpha^- L_-^\alpha + \Sigma_\alpha^- L_{\Sigma-}^\alpha + Z L_Z + \tilde{Z} L_{\tilde{Z}} \quad (5.9)$$

of this algebra satisfies MC equation $d\Omega + i\Omega \wedge \Omega = 0$,

$$\begin{aligned} dL_P^a + L_c^a L_P^c + i(\bar{L}_+^i \gamma^{a4} L_+^i) &= 0, & dL^{ab} + L^a L^b &= 0, \\ dL^\pi - L_{Bc} L_P^c - 2i(\bar{L}_+^i L_-^i) &= 0, & dL_B^a + L_c^a L_B^c &= 0, & dL_U &= 0, \\ dL_+^{i\alpha} + \frac{1}{4} L^{ab}(\gamma_{ab} L_+)^{i\alpha} + i L_U(\gamma^4 L_+)^{i\alpha} &= 0, \\ dL_-^{i\alpha} + \frac{1}{4} L^{ab}(\gamma_{ab} L_-)^{i\alpha} - i L_U(\gamma^4 L_-)^{i\alpha} + \frac{1}{2} L_B^a(\gamma_{a4} L_+)^{i\alpha} &= 0, \\ dL_{\Sigma-}^{i\alpha} + \frac{1}{4} L^{ab}(\gamma_{ab} L_{\Sigma-})^{i\alpha} - i L_U(\gamma^4 L_{\Sigma-})^{i\alpha} + \frac{1}{2} L_P^a(\gamma_{a4} L_+)^{i\alpha} &= 0, \\ dL_Z - i(\bar{L}_+^i \gamma^4 L_-^i) &= 0, & dL_{\tilde{Z}} - i(\bar{L}_+^i \gamma^4 L_{\Sigma-}^i) &= 0. \end{aligned} \quad (5.10)$$

The set of MC equations are consistent under the operation of "d" guaranteeing the closure of the superalgebra (5.4)-(5.8). In appendix C we present forms of L 's in a choice of coset parametrization though they are not used in the following.

Using the superalgebra G and a coset $G/H = G/(SO(3, 1) \times U(1))$ we will construct invariant 4-and 5-forms. H -invariant and closed 4-forms \tilde{K}_q , ($q = 0, \dots, 4$) which are reduced to the bosonic K_q in (2.10) are

$$\begin{aligned} \tilde{K}_0 &= \epsilon_{abcd} L_B^a L_B^b L_B^c L_B^d, \\ \tilde{K}_1 &= \epsilon_{abcd} \{L_P^a L_B^b - i(\bar{L}_+ \gamma^{ab} L_-)\} L_B^c L_B^d, \\ \tilde{K}_2 &= \epsilon_{abcd} \{L_P^a L_B^b - i(\bar{L}_+ \gamma^{ab} L_-)\} \{L_P^c L_B^d - i(\bar{L}_+ \gamma^{cd} L_-)\}, \\ \tilde{K}_3 &= \epsilon_{abcd} \{L_P^a L_P^b - 2i(\bar{L}_+ \gamma^{ab} L_{\Sigma-})\} \{L_P^c L_B^d - i(\bar{L}_+ \gamma^{cd} L_-)\}, \\ \tilde{K}_4 &= \epsilon_{abcd} \{L_P^a L_P^b - 2i(\bar{L}_+ \gamma^{ab} L_{\Sigma-})\} \{L_P^c L_P^d - 2i(\bar{L}_+ \gamma^{cd} L_{\Sigma-})\} \end{aligned} \quad (5.11)$$

and

$$d\tilde{K}_q = 0, \quad (q = 0, 1, \dots, 4). \quad (5.12)$$

Here in order to construct closed \tilde{K}_3 and \tilde{K}_4 we need to introduce L_{Σ^-} associated to the supercharge Σ^- added in (5.7).

Similarly closed and invariant 5-forms which are reduced to the bosonic Ω_q in (2.11) are constructed as

$$\begin{aligned}
\tilde{\Omega}_0 &= \tilde{K}_0 L^\pi - 2i \epsilon_{abcd} L_B^a L_B^b L_B^c (\bar{L}_- \gamma^{d4} L_-), \\
\tilde{\Omega}_1 &= \tilde{K}_1 L^\pi - 2i \epsilon_{abcd} L_B^a \{L_B^b L_B^c - i(\bar{L}_+ \gamma^{bc} L_-)\} (\bar{L}_- \gamma^{d4} L_-), \\
\tilde{\Omega}_2 &= \tilde{K}_2 L^\pi - 2i \epsilon_{abcd} L_P^a (L_P^b L_B^c - 2i(\bar{L}_+ \gamma^{ab} L_-)) (\bar{L}_- \gamma^{d4} L_-) \\
&\quad - 16i L_Z (\bar{L}_+ \gamma^4 L_-) (\bar{L}_+ \gamma^4 L_-), \\
\tilde{\Omega}_3 &= \tilde{K}_3 L^\pi - i \epsilon_{abcd} L_P^a \{L_P^b L_P^c - 4i(\bar{L}_+ \gamma^{bc} L_{\Sigma^-})\} (\bar{L}_- \gamma^{d4} L_-) \\
&\quad - 2i \epsilon_{abcd} L_P^a \{L_P^b L_B^c - 3i(\bar{L}_+ \gamma^{bc} L_-)\} (\bar{L}_- \gamma^{d4} L_{\Sigma^-}) + \\
&\quad + i \epsilon_{abcd} L_B^a \{L_P^b L_B^c - i(\bar{L}_+ \gamma^{bc} L_-)\} (\bar{L}_{\Sigma^-} \gamma^{d4} L_{\Sigma^-}) \\
&\quad + \frac{1}{3} \epsilon_{abcd} \{2L_P^a (\bar{L}_- \gamma^{bcd} L_{\Sigma^-}) + L_B^a (\bar{L}_{\Sigma^-} \gamma^{bcd} L_{\Sigma^-})\} (\bar{L}_+ \gamma^4 L_-) \\
&\quad - 32i L_{\tilde{Z}} (\bar{L}_+ \gamma^4 L_-) (\bar{L}_+ \gamma^4 L_-), \\
\tilde{\Omega}_4 &= \tilde{K}_4 L^\pi - 4i \epsilon_{abcd} L_P^a \{L_P^b L_P^c - 4i(\bar{L}_+ \gamma^{bc} L_{\Sigma^-})\} (\bar{L}_- \gamma^{d4} L_{\Sigma^-}) \\
&\quad + 2i \epsilon_{abcd} L_P^a \{L_P^b L_B^c + 2i(\bar{L}_+ \gamma^{bc} L_-)\} (\bar{L}_{\Sigma^-} \gamma^{d4} L_{\Sigma^-}) \\
&\quad + 8 \epsilon_{abcd} L_B^a (\bar{L}_+ \gamma^{bc} L_{\Sigma^-}) (\bar{L}_{\Sigma^-} \gamma^{d4} L_{\Sigma^-}) + \frac{4}{3} \epsilon_{abcd} L_P^a (\bar{L}_+ \gamma^4 L_-) (\bar{L}_{\Sigma^-} \gamma^{bcd} L_{\Sigma^-}) \\
&\quad - 64i L_{\tilde{Z}} (\bar{L}_+ \gamma^4 L_-) (\bar{L}_+ \gamma^4 L_{\Sigma^-}), \tag{5.13}
\end{aligned}$$

and

$$d\tilde{\Omega}_q = 0, \quad (q = 0, 1, \dots, 4). \tag{5.14}$$

In order to have closed 5-form $\tilde{\Omega}_2$ we need the central charge Z and to get $\tilde{\Omega}_3$ and $\tilde{\Omega}_4$ we use the central charge \tilde{Z} as well as Σ^- .

These \tilde{K}_q and $\tilde{\Omega}_q$ go back to the bosonic ones K_q in (2.10) and Ω_q in (2.11) when the fermions are put to zero and are the supersymmetric extensions of the Galilean Galileon in (2.15). All \tilde{K}_q , ($q = 0, 1, 2, 3, 4$) are closed and are surface term. All invariant closed 5-forms $\tilde{\Omega}_q$, ($q = 0, 1, 2, 3, 4$) are CE non-trivial since the bosonic pieces are non-trivial. Thus $\tilde{\Omega}_q$, ($q = 0, 1, 2, 3, 4$) are used to construct five WZ Lagrangians of the supersymmetric model.

If we restrict ones which have bosonic body they are unique invariants, up to surface terms, as in the bosonic case. However there are other H -invariant, thus G -invariant, 4 and 5-forms which vanish when fermions are put to zero. For example a piece in \tilde{K}_1 in (5.11)

$$\epsilon_{abcd} i (\bar{L}_+ \gamma^{ab} L_-) L_B^c L_B^d, \tag{5.15}$$

is H -invariant 4-form. There are number of such invariant fermionic Lagrangians that could be added to the Lagrangian consistent with the supersymmetry.

6 Summary and Discussions

In this paper we have reexamined the cohomological structure of the Galileon models using MC equations of the Galileon algebras and understood how the Lagrangians appear

as invariant 4-forms and/or pseudo invariant WZ terms. As we can write the EOM in terms of MC forms we can understand why the inverse Higgs condition $L^\pi = 0$ appears from the H -covariance. It also manifests that the Galileon scalar π satisfies second order EOM. It is noticed that they are shown to hold without using particular parametrizations of the coset.

We have also proposed a supersymmetric Galileon algebra that contains bosonic Galileon algebra and the $\mathcal{N} = 1$ superPoincaré algebra as its subalgebras. We have constructed supersymmetric counterparts of the invariant and closed 4-forms and 5-forms of the Galilean Galileon. The former are surface term and the latter are used to construct the WZ Lagrangians. If we restrict ones which have bosonic body, that does not vanish when fermions are put to zero, they are unique ones. However there is an ambiguity of H -invariant 4-forms which vanish when fermions are put to zero.

There are several issues to be discussed further for establishing the supersymmetric Galileon theory. In constructing the supersymmetric Lagrangians we take pullback of the MC forms. There are two options one is pullback to 4-dimensional Minkowski space with coordinates x^μ and other is pullback to $\mathcal{N} = 1$ superspace with coordinates (x^μ, θ_+^α) . In the former case fields appears as

$$\pi(x), v^a(x), \theta_\pm(x), \dots \quad (6.1)$$

In the latter case fields appear as superfields,

$$\pi(x, \theta_+), v^a(x, \theta_+), \theta_-(x, \theta_+), \dots \quad (6.2)$$

The superfield $\pi(x, \theta_+)$ when expanded by fermionic coordinates θ_+ defines the Galileon supermultiplet. The (super)transformations are non-linearly realized on these fields following to (2.8). It is necessary to write down the Lagrangian and clarify nature of dynamical fields and auxiliary fields.

In the bosonic case the inverse Higgs condition $L^\pi = 0$ is derived as the EOM. Although it is concluded from the covariance in the bosonic case, it is not clear for the supersymmetric case since we can construct fermion bi-linear covariants which could appear in the variations. It is important that the EOM is solved for the broken boost variables v^a algebraically for the Galileon scalar satisfying second order EOM. (There is an option to impose the inverse Higgs condition[23] to reduce the boost variables v^a .)

If the model is considered as a relativistic 3-brane it is natural that the supersymmetric model possess kappa symmetries. However the 3-brane in 4-dimensions is not dynamical, filled in whole space-time, and the superGalileon appears as supersymmetric field theory in 4-dimensions the role of kappa symmetries is not evident. Both the kappa invariance and the appearance of IH condition depend on the choice of Lagrangian. It is interesting to examine if the kappa symmetry can be satisfied by fixing the above mentioned ambiguity of fermionic H -invariant Lagrangian terms.

It is also interesting to make clear the relation to other approaches of the supersymmetric Galileons. For example in [17] four dimensional (conformal) Galileon Lagrangians are supersymmetrized using chiral superfield, while we consider ones from reduction of five dimensional algebra. As bosonic Galileons are understood from higher dimensions we

expect the superGalileon is derived in the same way naturally. These remaining issues are discussed in future investigations.

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A General properties

Here we consider general graded Lie algebra,

$$[G_A, G_B] = i f_{AB}^C G_C, \quad f_{AB}^C = -(-)^{AB} f_{BA}^C, \quad (\text{A.1})$$

where we use A, B, \dots for even and odd generators of G . $(-)^{AB} = -1$ only when both G_A and G_B are odd. The MC one form is

$$\Omega = -ig^{-1}dg = G_A L^A, \quad (\text{A.2})$$

and MC equation is

$$d\Omega + i\Omega^2 = 0, \quad dL^A + \frac{1}{2} f_{BC}^A L^C L^B = 0. \quad (\text{A.3})$$

The consistency is equivalent to hold the Jacobi identity

$$0 = f_{BC}^A \{ (dL^C) L^B - L^C (dL^B) \} \rightarrow f_{BC}^A f_{DE}^C L^E L^D L^B = 0. \quad (\text{A.4})$$

Using coset coordinates Z^M of the G/H , the MC form components L^A are expressed as $L^A = dZ^M L_M^A(Z)$, and the MC equation becomes

$$dL^A + \frac{1}{2} f_{BC}^A L^C L^B = -dZ^M dZ^N \partial_N L_M^A + \frac{1}{2} f_{BC}^A dZ^M L_M^C dZ^N L_N^B = 0, \quad (\text{A.5})$$

where ∂_M is the left derivative with respect to Z^M . Then it holds

$$\partial_M L_N^A - (-)^{MN} \partial_N L_M^A + \frac{1}{2} f_{BC}^A ((-)^{NC} L_M^C L_N^B - (-)^{MN+MC} L_N^C L_M^B) = 0. \quad (\text{A.6})$$

We define $[\delta Z]^A$ by replacing dZ^M with δZ^M in L^A ,

$$[\delta Z]^A = \delta Z^M L_M^A. \quad (\text{A.7})$$

Using (A.6) the variation of L^A , under *any* variation δZ^M , is computed as

$$\delta L^A = (d\delta Z^M) L_M^A + dZ^M \delta Z^N \partial_N L_M^A = d[\delta Z]^A + f_{BC}^A L^C [\delta Z]^B. \quad (\text{A.8})$$

Remember this formula holds for any graded algebras in this ordering.

B Explicit parametrization of coset

Here we solve the IH equation $L^\pi = 0$ in (3.10) and express MC forms in terms of Galileon fields.

The explicit form of the MC form L 's depends on the parametrization of the coset. We parametrize the coset $G/SO(p, 1)$, for example, as

$$g = e^{iP_a x^a} e^{iP_\pi \pi} e^{iM_{a\pi} v^a} = g_0 e^{iP_\pi \pi} e^{iM_{a\pi} v^a}, \quad g_0 \equiv e^{iP_a x^a}. \quad (\text{B.1})$$

We first compute $\Omega_0 = -ig_0 dg_0$,

$$-ig_0^{-1} dg_0 = -i e^{-iP_a x^a} d e^{iP_a x^a} = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}, \quad (\text{B.2})$$

with

$$e^a = dx^a + O(x)^a{}_b dx^b \left(\frac{\text{sh}(\frac{X}{R})}{\frac{X}{R}} - 1 \right), \quad \omega^{ab} = \frac{dx^{[a} x^{b]}}{R^2} \frac{(\text{ch}(\frac{X}{R}) - 1)}{(\frac{X}{R})^2}, \quad (\text{B.3})$$

where $X = \sqrt{x_a x^a}$ and $O_a{}^b(x) = \delta_a{}^b - \frac{x_a x^b}{X^2}$. e^a and ω^{ab} are viel-bein and spin connection verifying the AdS MC equations,

$$de^a + \omega^{ab} e_b = 0, \quad d\omega^{ab} + \omega^{ac} \omega_c{}^b + \frac{1}{R^2} e^a e^b = 0. \quad (\text{B.4})$$

The full left invariant MC 1-forms are

$$\begin{aligned} L^a &= \left(e^a + \frac{v^a (e^b v_b)}{V^2} \left(\cos\left(\frac{V}{\omega}\right) - 1 \right) \right) \text{ch}\left(\frac{\pi}{R\omega}\right) - d\pi \frac{v^a}{\omega V} \sin\left(\frac{V}{\omega}\right), \\ L^{ab} &= \omega^{ab} + \frac{Dv^{[a} v^{b]}}{V^2} \left(\cos\left(\frac{V}{\omega}\right) - 1 \right) - e^{[a} v^{b]} \frac{1}{RV} \sin\left(\frac{V}{\omega}\right) \text{sh}\left(\frac{\pi}{R\omega}\right), \\ L^\pi &= d\pi \cos\left(\frac{V}{\omega}\right) + (e^b v_b) \frac{\omega}{V} \sin\left(\frac{V}{\omega}\right) \text{ch}\left(\frac{\pi}{R\omega}\right), \\ L^{a\pi} &= Dv^a + O^a{}_b(v) Dv^b \left(\frac{\omega}{V} \sin\left(\frac{V}{\omega}\right) - 1 \right) + \left(e^a + O^a{}_b(v) e^b \left(\cos\left(\frac{V}{\omega}\right) - 1 \right) \right) \frac{\omega}{R} \text{sh}\left(\frac{\pi}{R\omega}\right), \end{aligned} \quad (\text{B.5})$$

where $V = \sqrt{v_a v^a}$, $O_a{}^b(v) = \delta_a{}^b - \frac{v_a v^b}{V^2}$ and $Dv^a = dv^a + \omega^{ab} v_b$.

We solve the Higgs constraint for each Galileon cases.

1) Galilean brane (Galileon) $[\omega \rightarrow \infty, R \rightarrow \infty]$

The Higgs equation (3.10) is solved as,

$$L^\pi = d\pi + dx^a v_a = d\sigma^\mu (\partial_\mu \pi + e_\mu{}^a v_a) = 0, \quad \rightarrow \quad v_a = -e_a{}^\mu \partial_\mu \pi, \quad (\text{B.6})$$

where σ^μ , $(\mu = 0, 1, 2, 3)$ are parameters of the 3-brane, $e_\mu{}^a = \partial_\mu x^a$ is viel-bein and $e_a{}^\mu$ is its inverse. Using it

$$\begin{aligned} L^a &= dx^a = d\sigma^\mu e_\mu{}^a, \quad L^{ab} = \omega^{ab} = 0. \\ L^{a\pi} &= dv^a = d\sigma^\mu \partial_\mu v^a = -d\sigma^\mu e^{a\nu} \nabla_\nu \nabla_\mu \pi, \end{aligned} \quad (\text{B.7})$$

where the covariant derivative ∇_μ is with respect to the induced metric $g_{\mu\nu} = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$. If we take a static gauge $x^a = \sigma^a$, $e_\mu^a = \delta_\mu^a$ it becomes

$$v_a = -\partial_a \pi, \quad L^a = dx^a, \quad L^{a\pi} = -dx^b \partial^a \partial_b \pi. \quad (\text{B.8})$$

2) Poincaré brane (DBI Galileon) $[\omega = 1, R \rightarrow \infty]$

In this limit the Higgs equation is solved as,

$$L^\pi = d\pi \cos(V) + (e^a v_a) \frac{\sin V}{V} = 0, \quad \rightarrow \quad \tilde{v}_a \equiv v_a \frac{\tan V}{V} = -e_a^\mu \partial_\mu \pi. \quad (\text{B.9})$$

where $V = \sqrt{v^2}$, $\tilde{V} = \sqrt{\tilde{v}^2} = \tan V$, $(\partial\pi)^2 = g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$ and

$$\begin{aligned} L^a &= \left(e^a + \frac{v^a (e^b v_b)}{V^2} (\cos V - 1) \right) - d\pi \frac{v^a}{V} \sin V \\ &= d\sigma^\nu e^{a\nu} \left(g_{\mu\nu} + \frac{\partial_\mu \pi \partial_\nu \pi}{(\partial\pi)^2} (\sqrt{1 + (\partial\pi)^2} - 1) \right), \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} L^{a\pi} &= dv^a + O^a_b(v) dv^b \left(\frac{\sin V}{V} - 1 \right) \\ &= d\sigma^\mu e^{a\mu} \left(-\frac{\nabla_\nu \nabla_\mu \pi}{\sqrt{1 + (\partial\pi)^2}} + \frac{\partial_\nu \pi \partial_\mu (\partial\pi)^2}{2(\partial\pi)^2 (1 + (\partial\pi)^2)} (\sqrt{1 + (\partial\pi)^2} - 1) \right). \end{aligned} \quad (\text{B.11})$$

In the static gauge $x^a = \sigma^a$, $e_\mu^a = \delta_\mu^a$ and $\nabla_\mu = \partial_\mu$.

3) NH brane $[\omega \rightarrow \infty, R \text{ finite}]$

$$\begin{aligned} L^\pi &= d\pi + (e^a v_a) = d\sigma^\mu (\partial_\mu \pi + e_\mu^a v_a) = 0, \quad \rightarrow \quad v_a = -e_a^\mu \partial_\mu \pi, \\ L^a &= e^a = d\sigma^\mu e_\mu^a, \\ L^{a\pi} &= Dv^a + e^a \frac{\pi}{R^2} = -d\sigma^\mu e^{a\mu} \left(\nabla_\nu \nabla_\mu \pi - g_{\nu\mu} \frac{\pi}{R^2} \right) \end{aligned} \quad (\text{B.12})$$

where $D_\mu v^a = -e^{a\nu} \nabla_\nu \nabla_\mu \pi$ and the covariant derivative ∇_μ is with respect to the AdS_4 metric $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$.

4) AdS brane $[\omega = 1, R \text{ finite}]$

For AdS Galileon $L^\pi = 0$ is solved as

$$L^\pi = d\pi \cos(V) + \frac{(e^\nu v_\nu)}{V} \sin(V) \text{ch}\left(\frac{\pi}{R}\right) = 0, \quad \rightarrow \quad \tilde{v}_a \equiv v_a \frac{\tan V}{V} = -e_a^\mu \partial_\mu \Pi, \quad (\text{B.13})$$

where $\tan(\frac{\Pi}{2R}) = \tanh(\frac{\pi}{2R})$, and

$$\begin{aligned} L^a &= d\sigma^\nu e^{a\mu} \left(g_{\mu\nu} + \frac{\partial_\mu \Pi \partial_\nu \Pi}{(\partial\Pi)^2} (\sqrt{1 + (\partial\Pi)^2} - 1) \right) \frac{1}{\cos(\frac{\Pi}{R})}, \quad (\partial\Pi)^2 = g^{\mu\nu} \partial_\mu \Pi \partial_\nu \Pi \\ L^{a\pi} &= d\sigma^\mu e^{a\mu} \left(-\frac{\nabla_\nu \nabla_\mu \Pi}{\sqrt{1 + (\partial\Pi)^2}} + \frac{\partial_\nu \Pi \partial_\mu (\partial\Pi)^2}{2(\partial\Pi)^2 (1 + (\partial\Pi)^2)} (\sqrt{1 + (\partial\Pi)^2} - 1) \right) \\ &+ d\sigma^\mu e^{a\mu} \left(\frac{g_{\mu\nu}}{\sqrt{1 + (\partial\Pi)^2}} - \frac{\partial_\mu \Pi \partial_\nu \Pi}{(\partial\Pi)^2} \left(\frac{1}{\sqrt{1 + (\partial\Pi)^2}} - 1 \right) \right) \frac{\tan(\frac{\Pi}{R})}{R}. \end{aligned} \quad (\text{B.14})$$

In $R \rightarrow \infty$ and/or $\omega \rightarrow \infty$ limits they go to ones of contracted results.

For the conformal Galileon in section 4 we parametrize the coset $G/SO(p, 1)$ as

$$g = e^{iP_a x^a} e^{iD\pi} e^{iK_a v^a} \quad (\text{B.15})$$

the MC forms are

$$\begin{aligned} L_P^a &= e^\pi dx^a, & L_K^a &= dv^a + v^a d\pi - v^2 e^\pi dx^a + 2 e^\pi v^a (v dx), \\ L_D &= d\pi + 2 e^\pi (v dx), & L^{ab} &= 2 e^\pi v^{[a} dx^{b]}. \end{aligned} \quad (\text{B.16})$$

Solving $L_D = 0$ as

$$v^a = -\frac{1}{2} e_a^a \partial_a \pi, \quad e_\mu^a = e^\pi \partial_\mu x^a, \quad (\text{B.17})$$

v^a is eliminated in terms of the conformal Galileon field π and

$$L_P^a = d\sigma^\mu e_\mu^a, \quad L_K^a = -\frac{1}{2} d\sigma^\mu e^{a\nu} \left(\nabla_\mu \nabla_\nu \pi + \frac{1}{2} g_{\mu\nu} (\partial\pi)^2 \right), \quad (\text{B.18})$$

where ∇_μ is with respect to the conformal metric $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$. The EOM for the Galileon π is given from (4.9) using (B.18).

C Supercoset $G/(SO(3, 1) \times U(1))$

The explicit form of the MC form L 's depends on the parametrization of the coset. For the supercoset $G/(SO(3, 1) \times U(1))$ for the superalgebra G in (5.4)-(5.8) we parametrize the coset element g , for example, as

$$g = e^{iP_a x^a} e^{iP_\pi \pi} e^{iQ^+ \theta_+} e^{iQ^- \theta_-} e^{i\Sigma^- \xi_-} e^{iB_a v^a} e^{iZc} e^{i\tilde{Z}\tilde{c}}. \quad (\text{C.1})$$

where θ_\pm and ξ_- are odd coordinates of the coset associated to Q^\pm and Σ^- and c, \tilde{c} are even scalar coordinates of the central charges Z, \tilde{Z} . The left invariant MC form $\Omega = -g^{-1} dg$ is computed as

$$\Omega = P_a L_P^a + \frac{1}{2} M_{ab} L^{ab} + P_\pi L^\pi + B_a L_B^a + Q_\alpha^+ L_+^\alpha + Q_\alpha^- L_-^\alpha + \Sigma_\alpha^- L_{\Sigma-}^\alpha + Z L_Z + \tilde{Z} L_{\tilde{Z}}, \quad (\text{C.2})$$

where

$$\begin{aligned} L_P^a &= dx^a - i \bar{\theta}_+^i \gamma^{a4} d\theta_+^i, & L_B^a &= dv^a, & L^{ab} &= 0, & L_U &= 0, \\ L^\pi &= d\pi - 2i \bar{\theta}_-^i d\theta_+^i + v_a (dx^a - i \bar{\theta}_+^i \gamma^{a4} d\theta_+^i), \\ L_+^i &= d\theta_+^i, & L_-^i &= d\theta_-^i - \frac{1}{2} \gamma_{a4} v^a d\theta_+^i, & L_{\Sigma-}^i &= d\xi_-^i + \frac{1}{2} \gamma_{a4} \theta_+^i dx^a, \\ L_Z &= dc + i \bar{\theta}_-^i \gamma^4 d\theta_+^i, & L_{\tilde{Z}} &= d\tilde{c} + i \bar{\xi}_-^i \gamma^4 d\theta_+^i. \end{aligned} \quad (\text{C.3})$$

They satisfy the MC equations (5.10) and are building blocks of the invariant forms Ω_q (5.13) for superGalileon WZ Lagrangians.

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